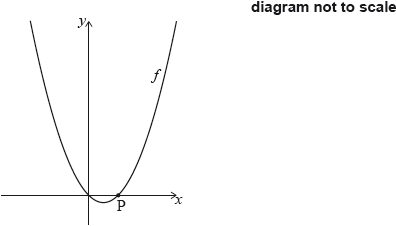
Calculus Assignment

Tangents and normal s and there equation

**1a.** *[3 marks]*

Let , for . The following diagram shows part of the graph of .



The graph of  crosses the -axis at the origin and at the point .

Show that .

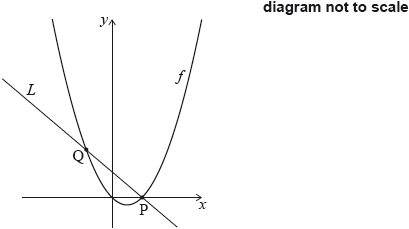
**1b.** *[3 marks]*

The line *L* is the normal to the graph of *f* at P.

Find the equation of  in the form .

**1c.** *[4 marks]*

The line  intersects the graph of  at another point Q, as shown in the following diagram.



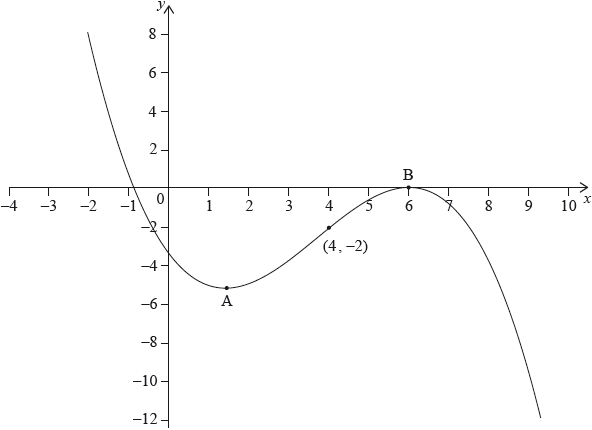
Find the -coordinate of Q.

**1d.** *[6 marks]*

Find the area of the region enclosed by the graph of  and the line .

**2a.** *[1 mark]*

The following diagram shows the graph of , the derivative of .



The graph of  has a local minimum at A, a local maximum at B and passes through .

The point  lies on the graph of the function, .

Write down the gradient of the curve of  at P.

**2b.** *[3 marks]*

Find the equation of the normal to the curve of  at P.

**2c.** *[2 marks]*

Determine the concavity of the graph of  when  **and** justify your answer.

**3a.** *[3 marks]*

A quadratic function  can be written in the form . The graph of  has axis of symmetry  and -intercept at 

Find the value of .

**3b.** *[3 marks]*

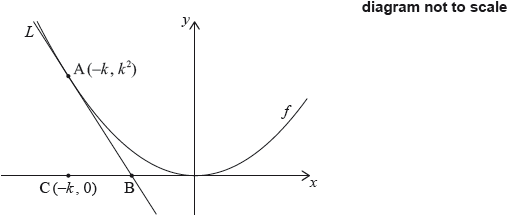
Find the value of .

**3c.** *[8 marks]*

The line  is a tangent to the curve of . Find the values of .

**4a.** *[1 mark]*

Let . The following diagram shows part of the graph of .



The line  is the tangent to the graph of  at the point , and intersects the -axis at point B. The point C is .

Write down .

**4b.** *[2 marks]*

Find the gradient of .

**4c.** *[5 marks]*

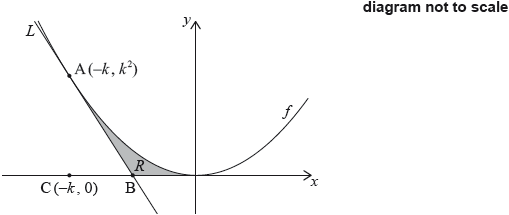
Show that the -coordinate of B is .

**4d.** *[2 marks]*

Find the area of triangle ABC, giving your answer in terms of .

**4e.** *[7 marks]*

The region  is enclosed by , the graph of , and the -axis. This is shown in the following diagram.



Given that the area of triangle ABC is  times the area of , find the value of .

**5a.** *[4 marks]*

Let , for .

Find .

**5b.** *[2 marks]*

Consider another function . Let R be a point on the graph of . The -coordinate of R is 1. The equation of the tangent to the graph at R is .

Write down .

**5c.** *[2 marks]*

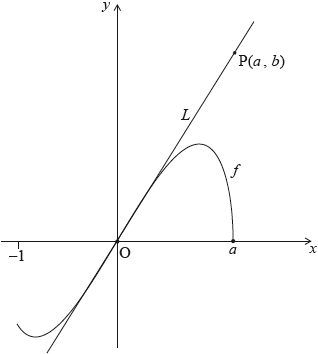
Find .

**5d.** *[7 marks]*

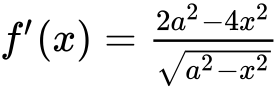
Let . Find the equation of the tangent to the graph of  at the point where .

**6a.** *[6 marks]*

The following diagram shows the graph of , for , where .



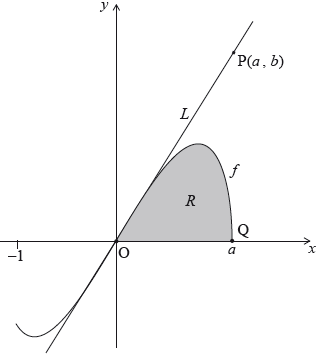
The line  is the tangent to the graph of  at the origin, O. The point  lies on .

(i) Given that , for , find the equation of .

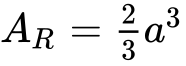
(ii) Hence or otherwise, find an expression for  in terms of .

**6b.** *[6 marks]*

The point  lies on the graph of . Let  be the region enclosed by the graph of  and the -axis. This information is shown in the following diagram.



Let  be the area of the region .

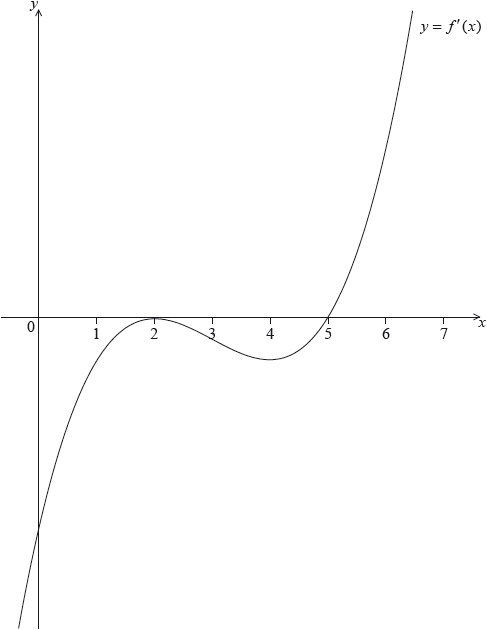
Show that .

**6c.** *[4 marks]*

Let  be the area of the triangle OPQ. Given that , find the value of .

**7a.** *[2 marks]*

Let , for  x  . The following diagram shows the graph of , the derivative of .



The graph of  has a local maximum when , a local minimum when , and it crosses the *-*axis at the point .

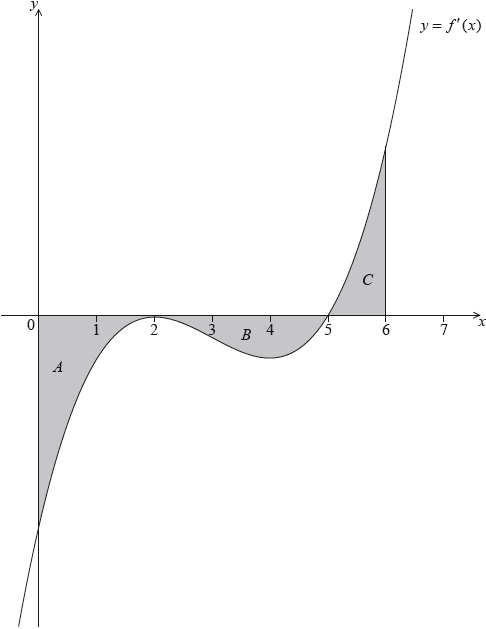
Explain why the graph of  has a local minimum when .

**7b.** *[2 marks]*

Find the set of values of  for which the graph of  is concave down.

**7c.** *[5 marks]*

The following diagram shows the shaded regions ,  and .



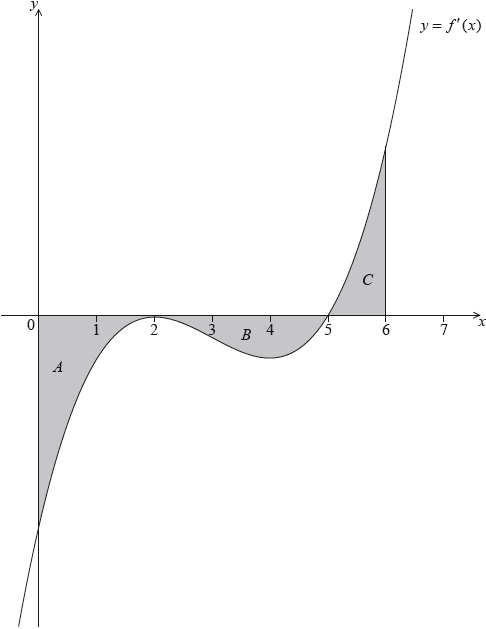
The regions are enclosed by the graph of , the -axis, the -axis, and the line .

The area of region  is 12, the area of region  is 6.75 and the area of region  is 6.75.

Given that , find .

**7d.** *[6 marks]*

The following diagram shows the shaded regions ,  and .



The regions are enclosed by the graph of , the *x*-axis, the *y*-axis, and the line .

The area of region  is 12, the area of region  is 6.75 and the area of region  is 6.75.

Let . Given that , find the equation of the tangent to the graph of  at the point where .

**8a.** *[2 marks]*

A function  has its derivative given by , where  is a constant.

Find .

**8b.** *[3 marks]*

The graph of  has a point of inflexion when .

Show that .

**8c.** *[2 marks]*

Find .

**8d.** *[4 marks]*

Find the equation of the tangent to the curve of  at , giving your answer in the form .

**8e.** *[3 marks]*

Given that , explain why the graph of  has a local maximum when .

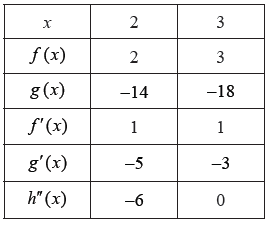
**9.** *[6 marks]*

Let . The line  is the tangent to the curve of  at .

Find the equation of  in the form .

**10a.** *[3 marks]*

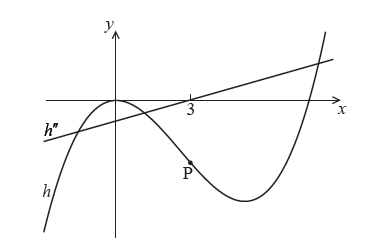
Consider the functions  ,  and  . The following table gives some values associated with these functions.



Write down the value of  , of  , and of  .

**10b.** *[2 marks]*

The following diagram shows parts of the graphs of  and  .



There is a point of inflexion on the graph of  at P, when  .

Explain why P is a point of inflexion.

**10c.** *[2 marks]*

Given that  ,

find the -coordinate of P.

**10d.** *[7 marks]*

find the equation of the normal to the graph of  at P.

**11a.** *[3 marks]*

Let  , for  .

Find  .

**11b.** *[3 marks]*

Let  be a quadratic function such that  . The line  is the axis of symmetry of the graph of  .

Find  .

**11c.** *[4 marks]*

The function  can be expressed in the form  .

(i) Write down the value of  .

(ii) Find the value of  .

**11d.** *[6 marks]*

Find the value of  for which the tangent to the graph of  is parallel to the tangent to the graph of  .

**12a.** *[1 mark]*

Let  .

Write down  .

**12b.** *[4 marks]*

The tangent to the graph of *f* at the point  has gradient *m* .

(i) Show that  .

(ii) Find *b* .

**12c.** *[1 mark]*

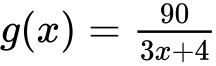
Hence, write down the equation of this tangent.

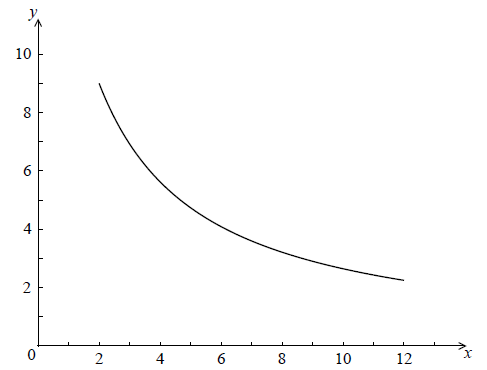
**13a.** *[4 marks]*

Let  . The line *L* is the tangent to the curve of *f* at (4, 6) .

Find the equation of *L* .

**13b.** *[6 marks]*

Let  , for  . The following diagram shows the graph of *g* .



Find the area of the region enclosed by the curve of *g* , the *x*-axis, and the lines  and  . Give your answer in the form  , where  .

**13c.** *[3 marks]*

The graph of *g* is reflected in the *x*-axis to give the graph of *h* . The area of the region enclosed by the lines *L* ,  ,  and the *x*-axis is 120  .

Find the area enclosed by the lines *L* ,  ,  and the graph of *h* .

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